

Appendix E

Plane Geometry

A. Circle

A circle is defined as a closed plane curve every point of which is equidistant from a fixed point within the curve.

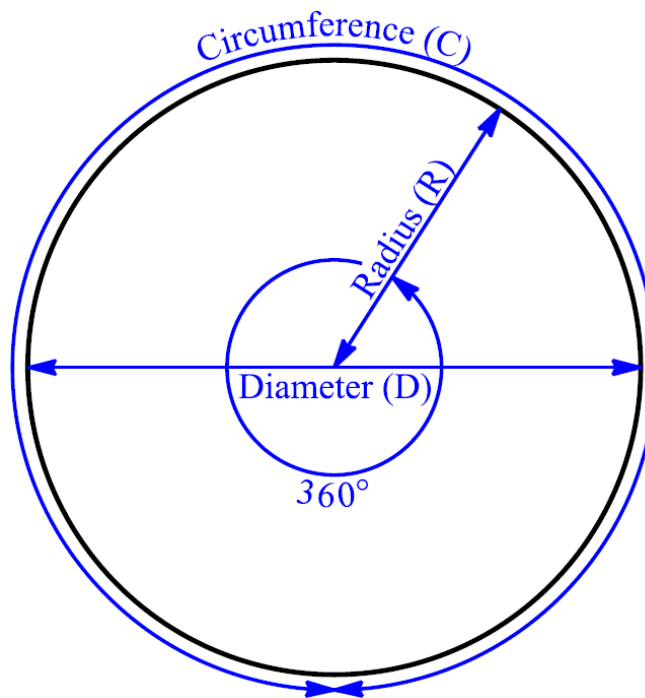


Figure E-1. Circle components.

1. Pi

In mathematics, Pi (π) is used to equate the circumference of a circle to its diameter. It is also used to relate the area of a circle to its radius. Pi has an approximate value of 3.14159265. This value is approximated because pi is an irrational number, which means that its value cannot be expressed exactly as a fraction. Consequently, its decimal representation never ends or repeats. Pi is used in many calculations involving area and volume, making it one of the most important mathematical constants.

2. Radians

The radian is the standard unit of angular measure used in many areas of mathematics. It describes the angle subtended at the center of a circle by an arc. The radian is a dimensionless value equal to the length of an arc divided by the radius of the arc. Refer to Figure E-2 for an illustration of a one radian angle.

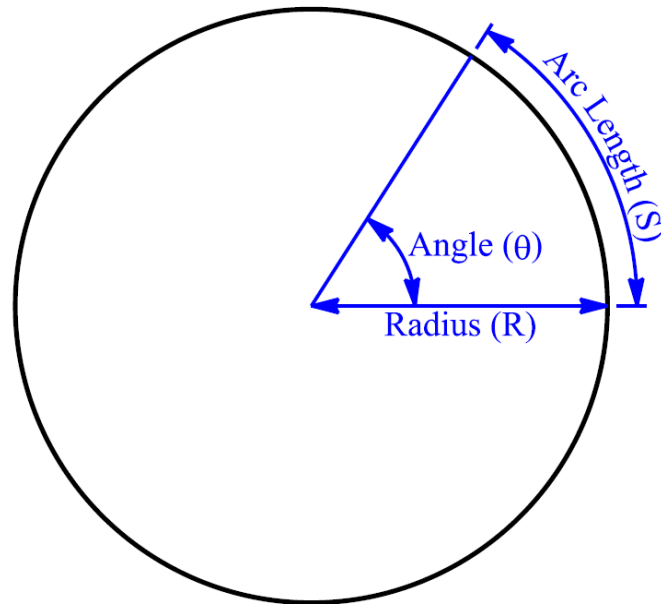


Figure E-2. A one radian angle.

The angle (θ) in radians is equal to the ratio of the arc length (S) to the radius of a circle (R):

$$\theta = S / R$$

If the arc length and radius are equal, then the angle (in radians) is equal to 1. To relate radians to degrees, the magnitude in radians of one complete revolution of a circle is the length of the circumference divided by the radius. The angle (θ) is equal 360° and the arc length (S) is equal to $2\pi R$. Using the radian equation:

$$\theta = S/R; \quad 360^\circ = 2\pi R/R = 2\pi$$

Therefore, an angle of 2π in radians is equal to 360 degrees, meaning that one radian is equal to $180/\pi$ degrees (approximately 57.29578°).

3. General Circular Formulas

The following equations are used to compute radius (R), diameter (D), circumference (C), arc length (S) and angle (θ). The angle can be expressed in terms of radians or degrees. Refer to Figures E-1 and E-2 for illustrations of these components.

$$D = 2R; \quad C = 2\pi R = \pi D$$

$$S = \theta R \text{ (angle in radians); } \quad S = \pi\theta R/180^\circ \text{ (angle in degrees)}$$

The following equations are used to compute the area of a circle, sector, and segment.

Area (A) of a circle formula:

$$A = \pi R^2 = \frac{1}{4}\pi D^2$$

The shaded area in Figure E-3 between the two radii (R) and the circular arc is a circular section.

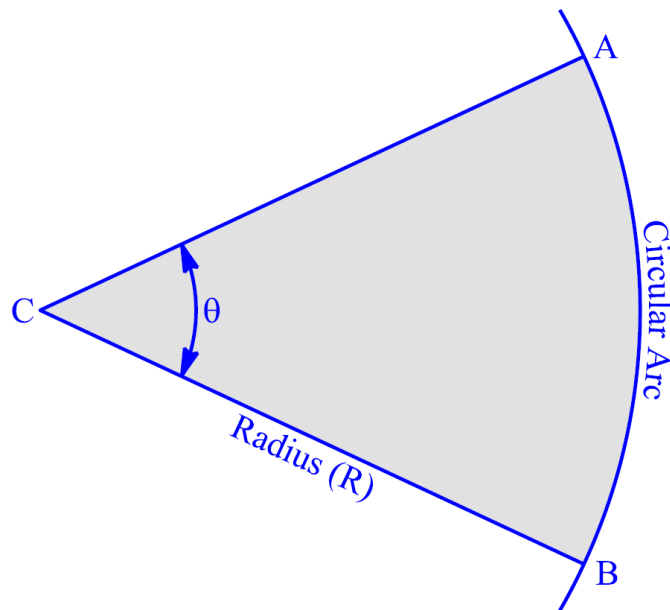


Figure E-3. Area of a sector.

Area of a section formula:

$$A = \frac{1}{2}\theta R^2 \text{ (angle in radians)}$$

$$A = \frac{\theta\pi R^2}{360^\circ} \text{ (angle in degrees)}$$

The shaded area in Figure E-4 between chord AB and the circular arc is a circular segment. The area is equal to the area of the circular sector minus the area of the triangular portion.

Area of a segment formula:

$$A = \frac{1}{2}\theta R^2 - \frac{1}{2}R^2 \sin \theta \text{ (angle in radians)}$$

$$A = \frac{\theta\pi R^2}{360^\circ} - \frac{1}{2}R^2 \sin \theta \text{ (angle in degrees)}$$

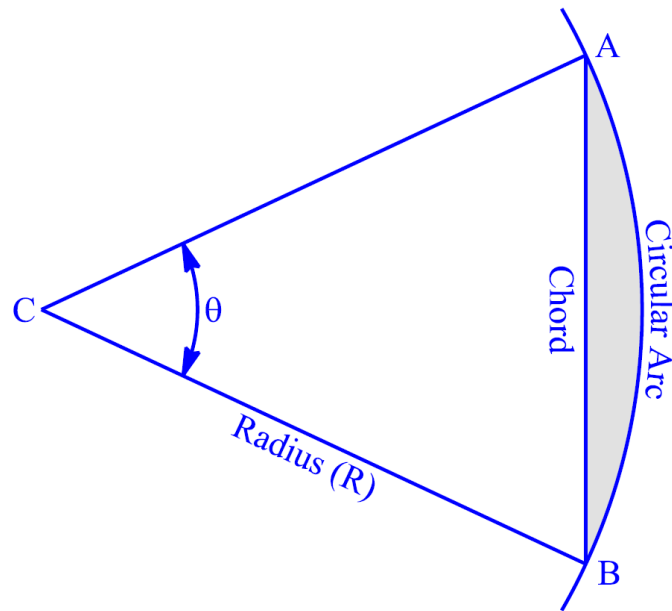


Figure E-4. Area of a segment.

B. Polygon

A polygon is a closed plane figure bounded by three or more straight line segments. An equilateral polygon is a polygon with all sides being the same length. An equiangular polygon is a polygon with equal interior angles. Any polygon that is both equilateral and equiangular is referred to as a regular polygon. An irregular polygon has sides of differing lengths and/or unequal interior angles.

1. General Polygon Formulas

The perimeter of a polygon is the sum of the lengths of the sides. The sum of the interior angles of a polygon is related to the number of sides (n).

$$Sum = (n - 2) * 180^\circ$$

The area (A) of a regular n-sided polygon is determined from the side length (s), apothem (a), and circumradius (r). The apothem is a line segment drawn from the center of the polygon to the midpoint of one of its sides. Regular polygons are the only polygons that have apothems. The circumradius is the distance from the center of the polygon to a circle which passes through all the vertices of the polygon.

$$r = \frac{s}{2 \sin(180^\circ/n)}; \quad a = r \cos \frac{180^\circ}{n}$$

$$A = \frac{1}{2}nsa = na^2 \tan \frac{180^\circ}{n} = \frac{1}{4}ns^2 \cot \frac{180^\circ}{n}$$

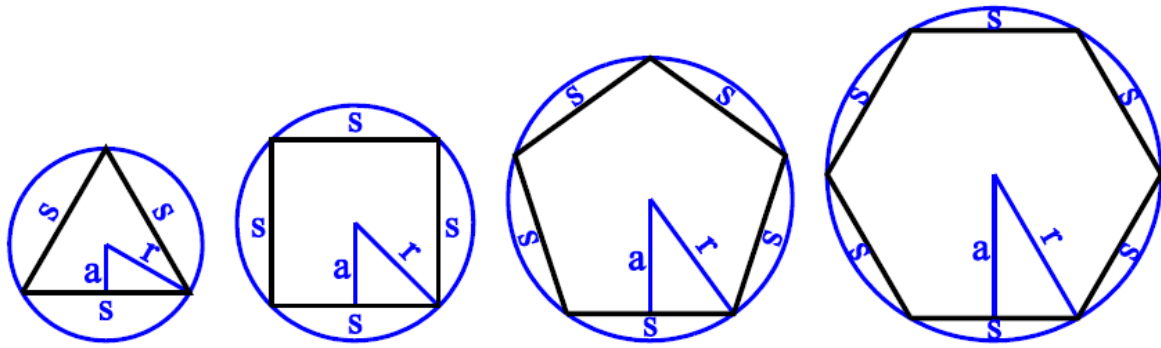


Figure E-5. Regular polygons.

2. Polygon Classifications

Polygons are primarily classified by the number of sides.

a. Triangle

A triangle is any polygon with three sides.

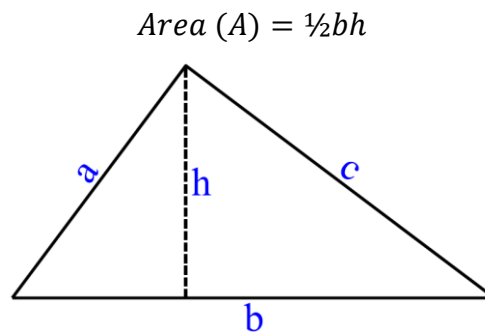


Figure E-6. Triangle components.

A 3:4:5 triangle is a right triangle whose sides are in the ratio 3:4:5. Multiply the sides by any number and the result will still be a right triangle whose sides are in the ratio 3:4:5 (e.g. 6:8:10 or 15:20:25). Refer to Figure E-7 for an illustration of a 3:4:5 triangle.

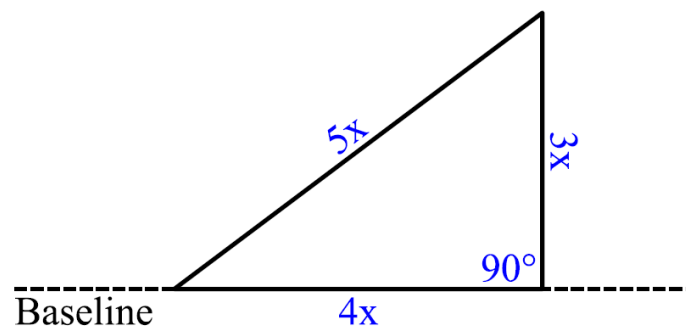


Figure E-7. A 3:4:5 triangle.

The 3:4:5 triangle may be used to lay out a line that is perpendicular to another line. For example, layout a distance along a baseline of 80 ft ($4 * 20$), then measure a distance out from the baseline of 60 ft ($3 * 20$). The line will be exactly perpendicular to the baseline when the diagonal distance is 100 ft ($5 * 20$).

b. Quadrilateral

A quadrilateral is any polygon with four sides. A parallelogram is a quadrilateral with opposite sides equal and parallel.

(1) Square

A square is a parallelogram with four sides of equal length and four right angles (90°).

$$\text{Area } (A) = b^2$$

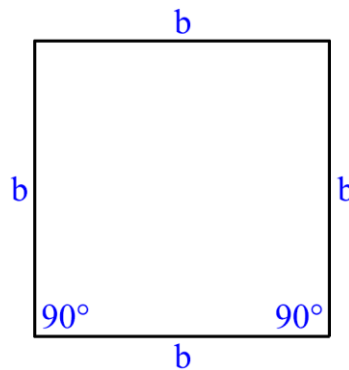


Figure E-8. Square components.

(2) Rectangle

A rectangle is a parallelogram with adjacent sides of unequal length and four right angles.

$$A = \frac{1}{2}bh$$

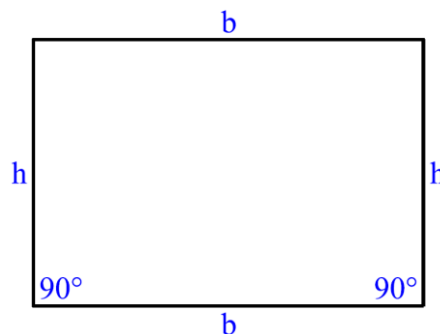


Figure E-9. Rectangle components.

(3) Rhombus

A rhombus is a parallelogram with four sides of equal length and four oblique angles. An oblique angle is defined as any angle that is not a right angle.

$$A = bh$$

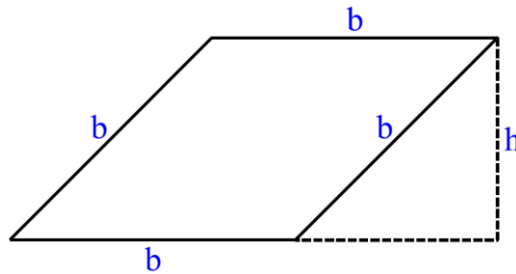


Figure E-10. Rhombus components.

(4) Rhomboid

A rhomboid is a parallelogram with adjacent sides of unequal length and four oblique angles.

$$A = bh$$

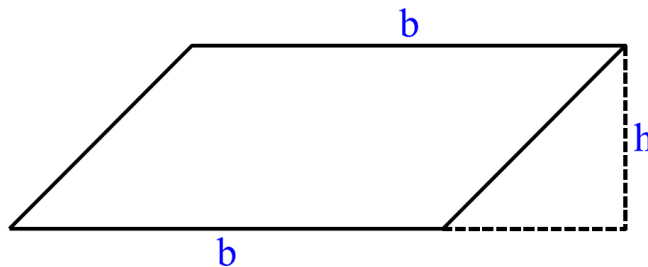


Figure E-11. Rhomboid components.

(5) Trapezoid

A trapezoid is a quadrilateral with two parallel sides.

$$A = \frac{1}{2}(b_1 + b_2)h$$

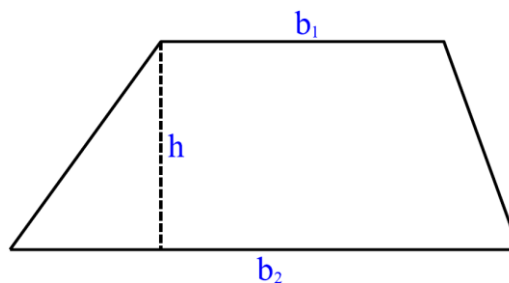


Figure E-12. Trapezoid components.

(6) Trapezium

A trapezium is a quadrilateral without any parallel sides.

$$A = \frac{1}{2}dh + [\frac{1}{4}b(h_1 + h_2)]$$

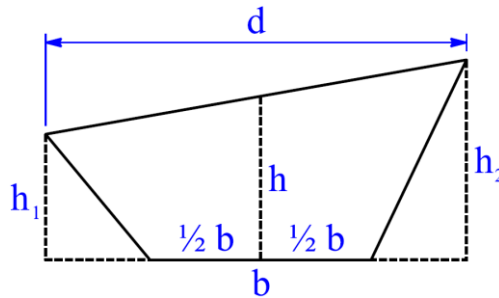


Figure E-13. Trapezium components.

c. Pentagon, Irregular

A pentagon is any polygon with five sides. The formula for calculating the area of an irregular pentagon is the same formula used to calculate the area of a trapezium.

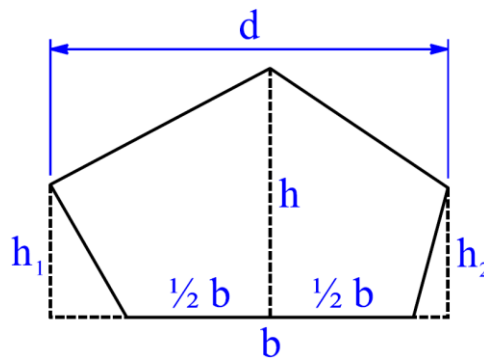


Figure E-14. Pentagon components.