Appendix F
Volume

A. Polyhedron

A polyhedron is a three-dimensional geometric solid with flat faces and straight edges. Polyhedrons are named according to the number of faces; a tetrahedron has 4, a pentahedron has 5, a hexahedron has 6, a heptahedron has 7, and an octahedron has 8.

A regular polyhedron is a symmetrical geometric figure with all faces being equal in size and shape. A parallelepiped is a hexahedron formed by six parallelograms, a four sided polygon with opposite sides equal and parallel.

1. Prism

In geometry, a prism is a polyhedron with two n-sided parallel polygonal bases and n other parallelogram sides joining the two bases. Prisms are named according to the number of sides of the base, (e.g. a prism with a five sided-base is called a pentagonal prism). A prism in which the joining faces and edges are perpendicular to each base is a right prism. If the joining faces and edges are not perpendicular to the bases, it is referred to as an oblique prism.

The volume of a prism is the product of the area of one base (B) multiplied by the perpendicular distance (h) between the two bases. Refer to Appendix E in this manual for more information on calculating the area of a polygon.

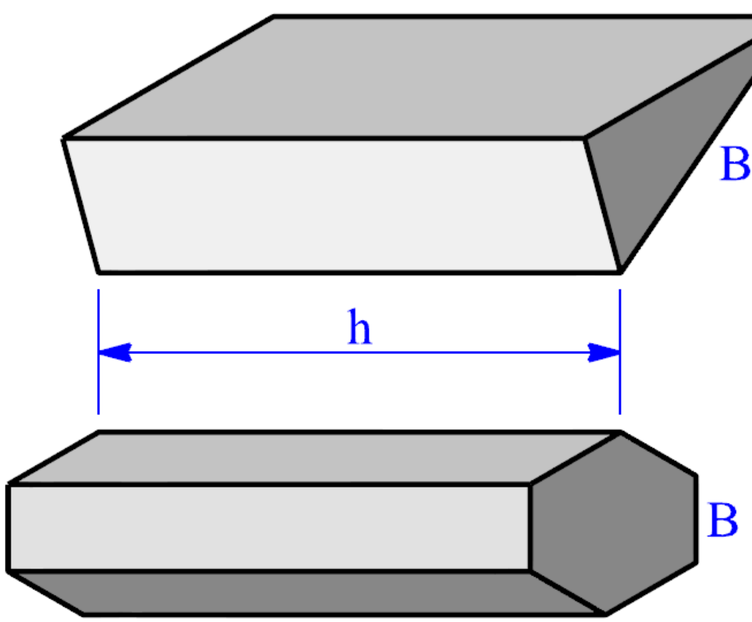

Figure F- 1. Two types of prisms.

a. Cube

A cube is an equal-sided right prism.

$$Volume\ (V) = Bh = (h * h) * h = h^3$$

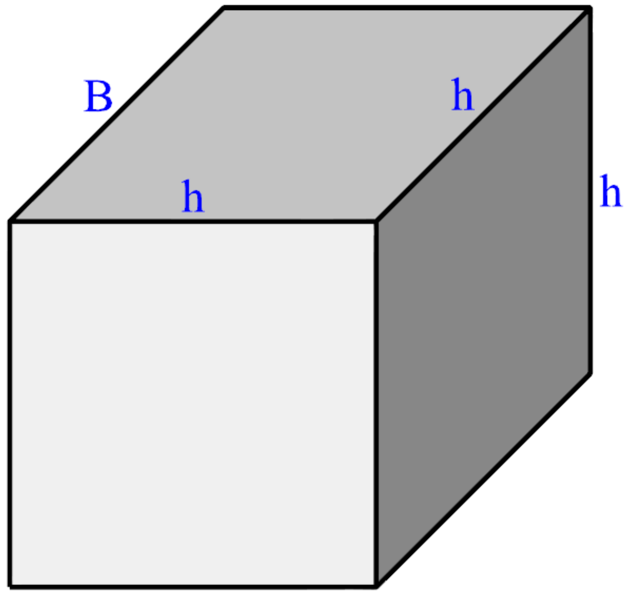

Figure F-2. An equal-sided right prism.

b. Cuboid

A cuboid is a right prism with adjacent joining faces of unequal size. A cuboid is also referred to as a right rectangular prism.

$$V = Bh = (a * b) * h$$

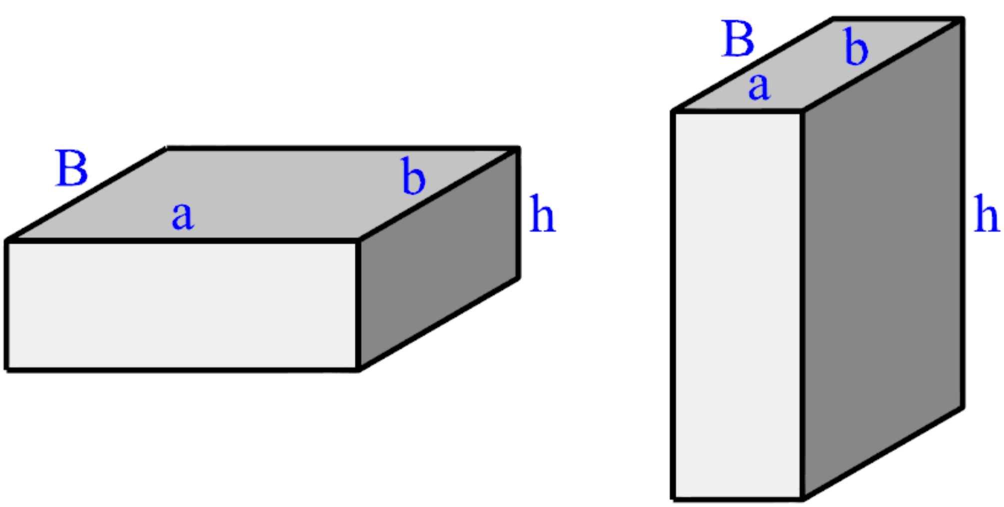

Figure F-3. Two right rectangular prisms.

c. Rhombohedron

A rhombohedron is an equal-sided oblique prism.

$$V = Bh = (b * b) * h$$

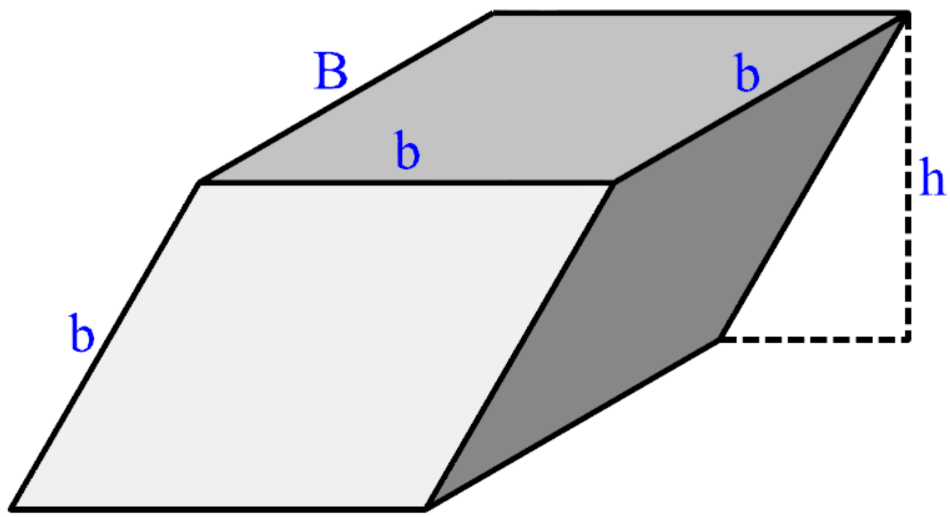

Figure F-4. An equal-sided oblique prism.

d. Rhombic Prism

A rhombic prism is a parallelepiped without 90° angles.

$$V = Bh = (a * b) * h$$

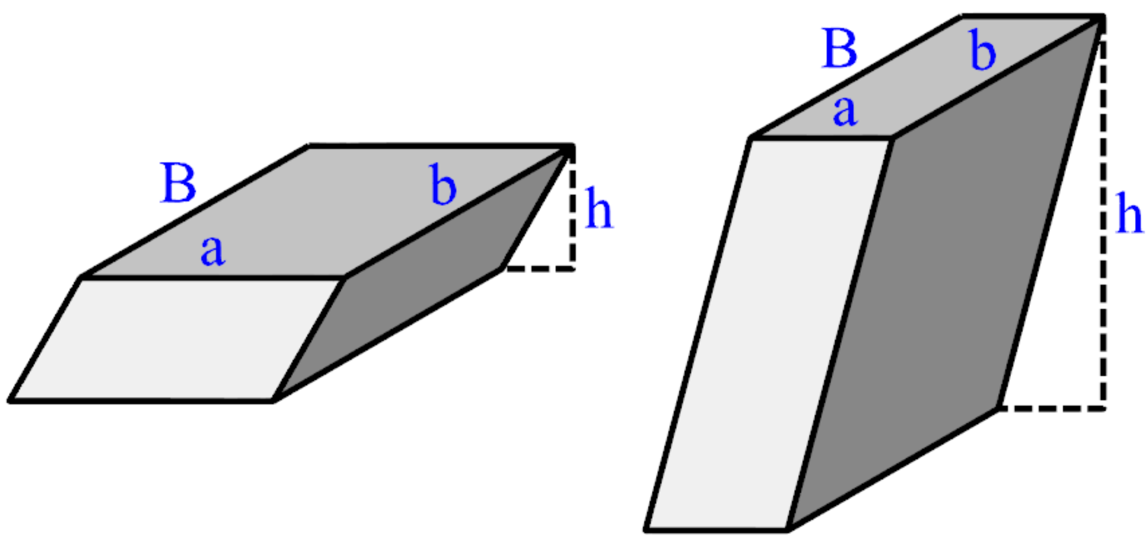

Figure F-5. Two rhombic prisms.

B. Cylinder

A cylinder is a three-dimensional geometric figure enclosed by a curvilinear surface and two parallel planes. The parallel planes, or bases, may be circular or elliptical in shape. A cylinder whose cross section is an ellipse is referred to as an elliptic cylinder. A right cylinder is formed when the surface is perpendicular to the bases. An oblique cylinder is formed when the two bases are offset from one another. The volume of a cylinder is calculated by multiplying the area of one base by the cylinder height.

$$V = Bh = \pi R^2 h$$

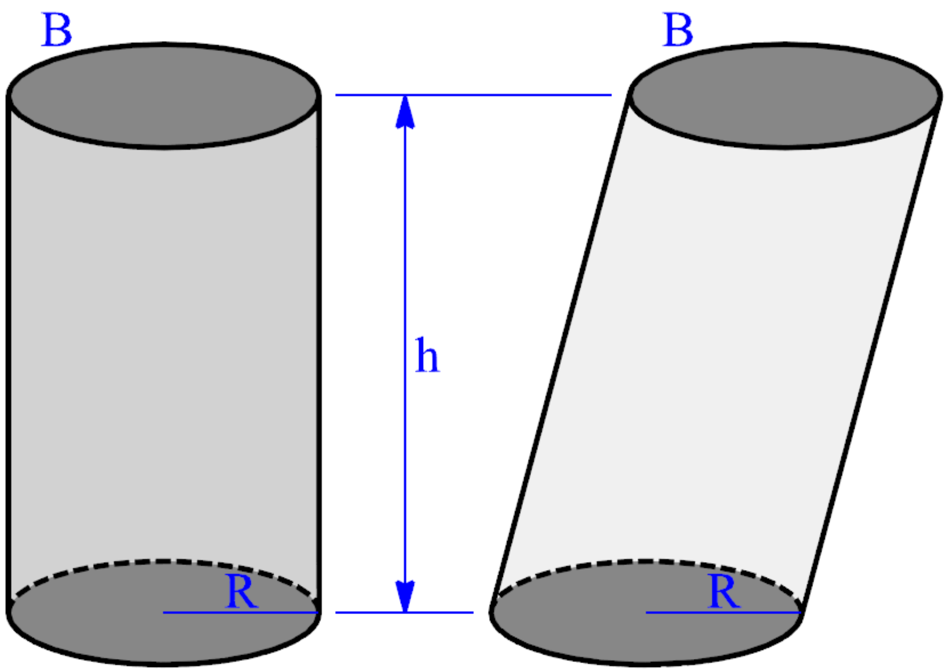

Figure F-6. A right cylinder and an oblique cylinder.

C. Cone

A cone is a three-dimensional geometric shape that tapers smoothly from a flat base to a single point called the apex (or vertex). The axis of a cone is a straight line running from the center of the base to the apex. When the axis is perpendicular to the base, then the cone is referred to as a right cone. An oblique cone is one in which the axis is not perpendicular to the base and the apex is offset. The volume of a cone is related to the area of the base and the perpendicular distance from the base to the apex.

$$V = {}^{1}/_{3}\, Bh = {}^{1}/_{3}\, \pi R^2 h$$

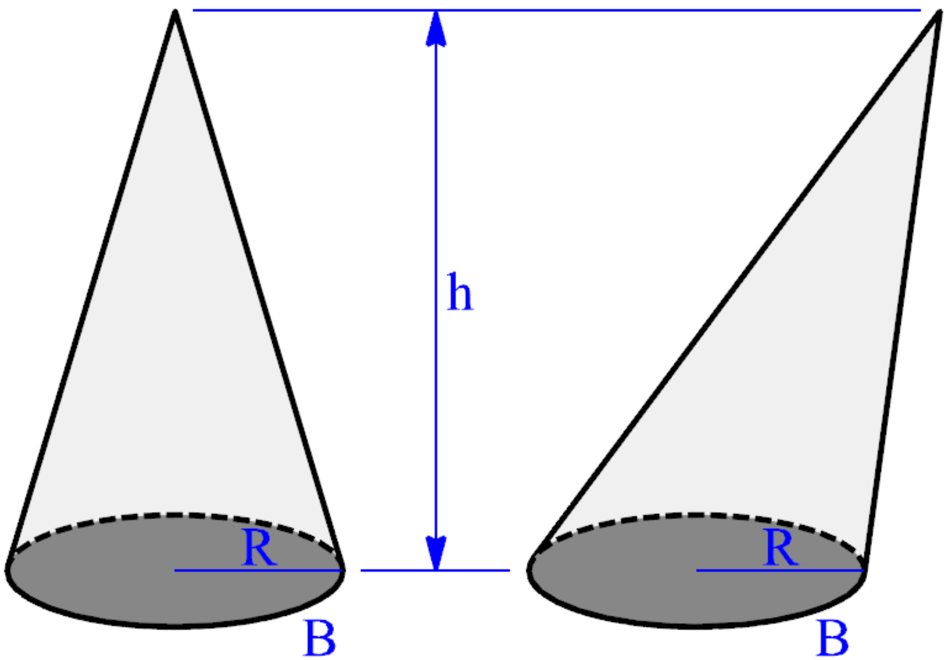

Figure F-7. A right cone and an oblique cone.

D. Pyramid

In geometry, a pyramid is a polyhedron formed by connecting a polygonal base and an apex. The joining surfaces of a pyramid are in the shape of a triangle. The volume of a pyramid is related to the area of the base and the perpendicular distance from the base to the apex. Refer to Appendix E in this manual for calculating the area of a polygon.

$$V = {}^{1}/_{3}\, Bh$$

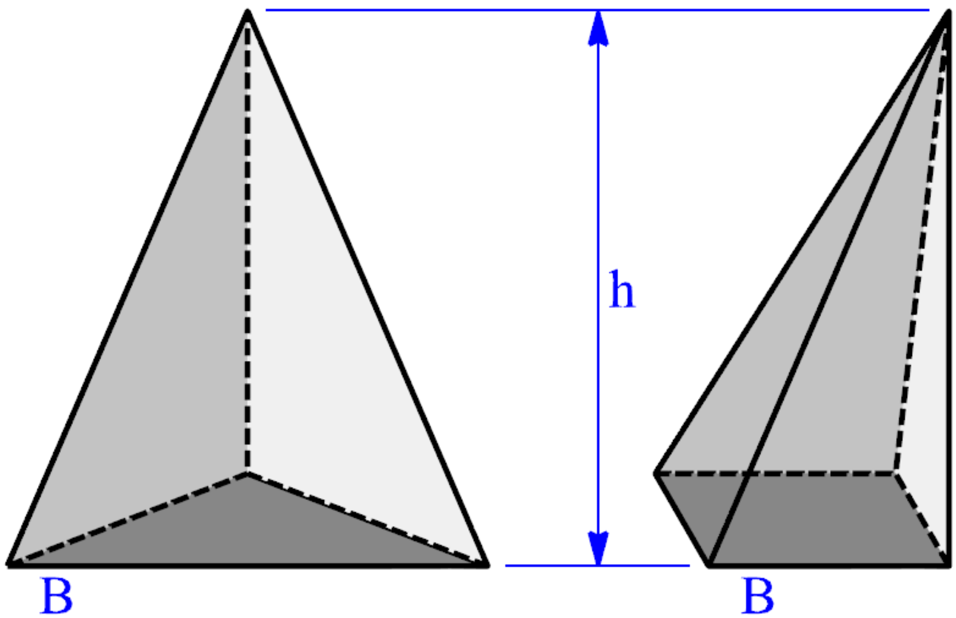

Figure F-8. Two pyramids.

E. Frustum

A frustum is the portion of a geometric figure, typically a cone or pyramid, truncated by a parallel plane. The volume of a frustum is related to the area of each base and the perpendicular distance between the two bases. Each base of the frustum will typically be a circle or a polygon. Refer to Appendix E in this manual for calculating the area of a circle or polygon.

$$V = {}^{1}\!/_{3}\, h * (B_1 + B_2 + \sqrt{B_1 * B_2}\,)$$

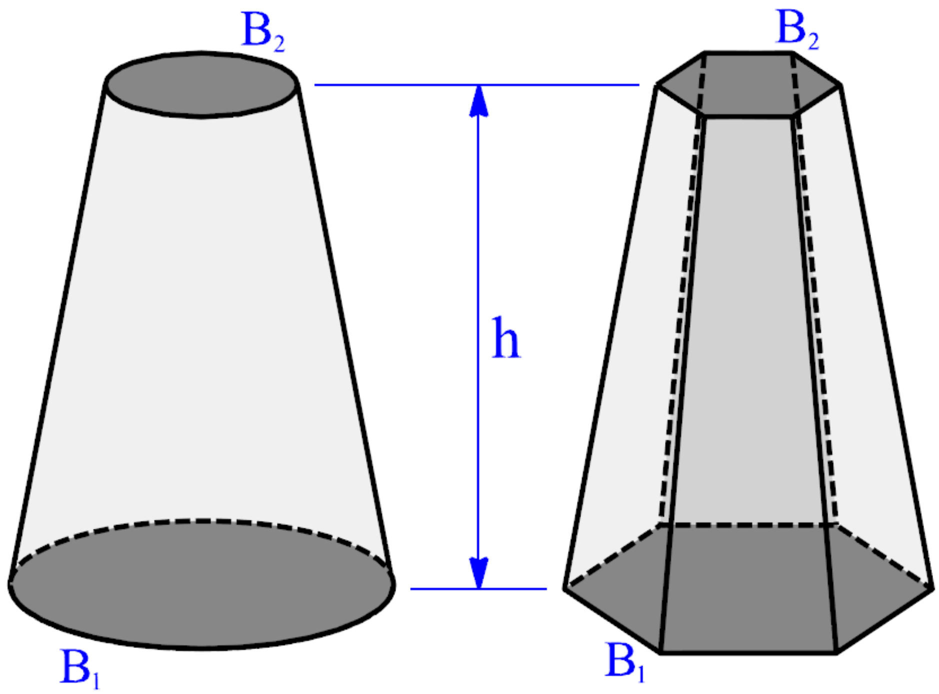

Figure F-9. Two types of frustums.

F. Sphere

A sphere is a symmetrical three-dimensional figure, whose surface is equidistant from a fixed point. The volume of a sphere is related to its radius (R).

$$V = {}^{4}/_{3}\,\pi R^3$$

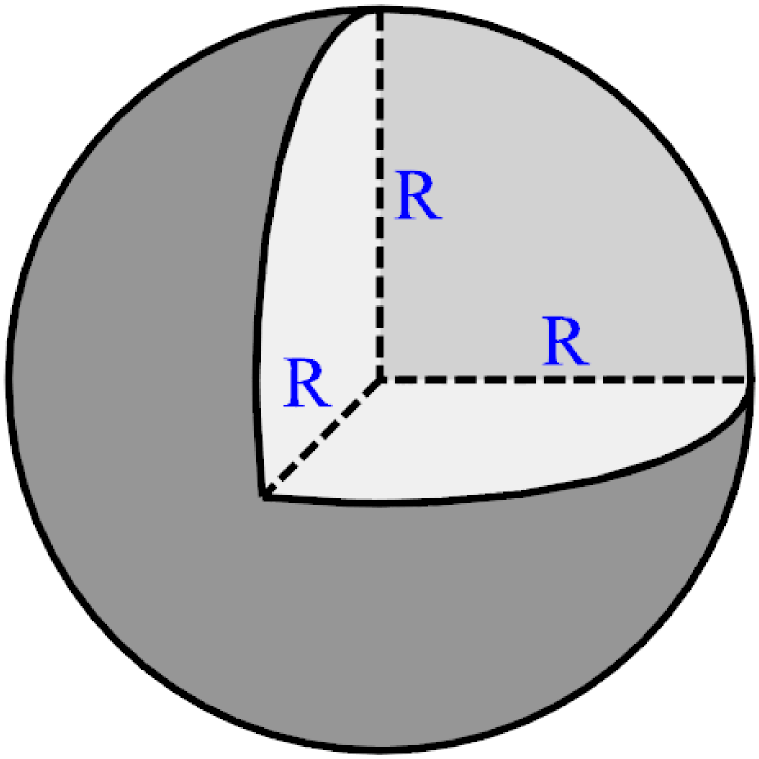

Figure F-10. Sphere

G. Earthwork

Cross-sections establish a profile of the existing ground perpendicular to a baseline at specific intervals, typically 50 to 100 ft. Cross-section measurements consist of an elevation and corresponding distance from a centerline. The existing ground is then plotted along with the proposed roadway template. The area of excavation (cut) or embankment (fill) at each cross-section is determined by comparing the existing and proposed surfaces. From the areas of cut or fill, earthwork volumes are can then be calculated.

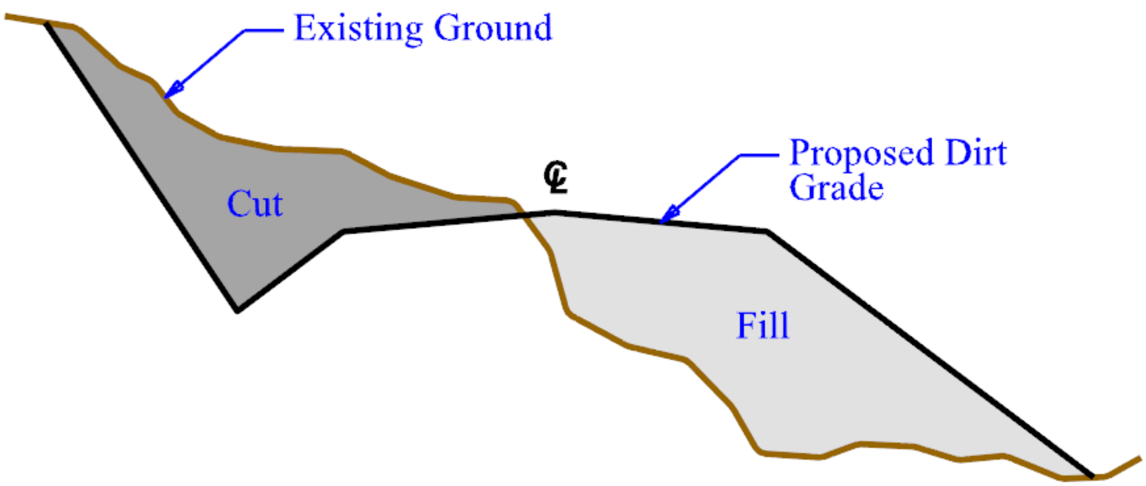

Figure F-11. Cross-section with cut and fill areas.

1. Average End-Area

As illustrated in Figure F-11, the cross-section represents the existing and proposed surfaces. The area between the two surfaces is referred to as the end-area. If the existing ground is above the proposed surface, the area in between is the amount of excavation. Conversely, if the existing ground is below the proposed surface, the area in between is the amount of embankment.

The volume (V) of earthwork (cut or fill) between adjacent cross-sections is obtained by taking multiplying the average of the end-areas (A_1 and A_2) by the distance (L) between the sections. Typically, the end-areas are in square feet and the distance between the sections is in feet. The corresponding volume (in cubic feet) is divided by 27 to obtain a volume in cubic yards.

$$V = {}^{1}/_{2}\,(A_1 + A_2) * L$$

This formula is considered exact only if the end areas (A_1 and A_2) are equal. However, it is only an approximation if the end areas are not equal. If one end-area has a value of zero, the earthwork volume is then similar to the volume of a pyramid.

$$V = {}^1/_3\, A * L$$

2. Prismoidal Method

The prismoidal method is used when a more exact method of determining earthwork volumes is warranted. A prismoid is a solid having ends that are parallel and sides that are trapezoidal. The ends and sides of the prismoid are not congruent, (i.e. they do not have the same size or shape). Most earthwork solids obtained from cross-sections fit this description.

The volume (V) of a prismoidal shape is calculated from the two end-areas (A_1 and A_2), the area (A_m) of a section midway between A_1 and A_2, and the distance (L) between the two outer sections.

$$V = {}^1/_6\,(A_1 + 4A_m + A_2) * L$$

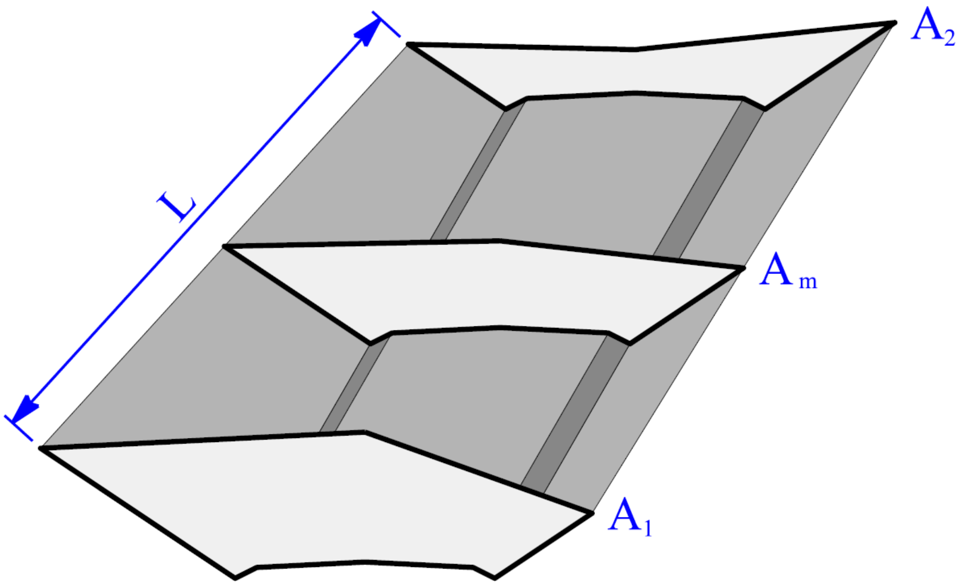

Figure F-12. Volume by prismoidal method.

The midpoint area is determined from averaging the corresponding linear heights and widths of the two end-areas and not by averaging their areas. As stated previously, the end-areas are in square feet and the distance between sections is in feet. The volume in cubic feet is divided by 27 to obtain the volume in cubic yards.

a. Prismoidal Correction

The difference between the volumes obtained by the average end-area method and the prismoidal method is referred to as the prismoidal correction (C_P). The correction formula is related to the distance (L) between two end-areas, the center heights (h) of an earthwork section (cut or fill) at the two end-areas, and the width (w) of an earthwork section (from slope intercept to slope intercept) at the two end-areas.

$$C_P = \frac{L * (h_1 - h_2)(w_1 - w_2)}{12}$$

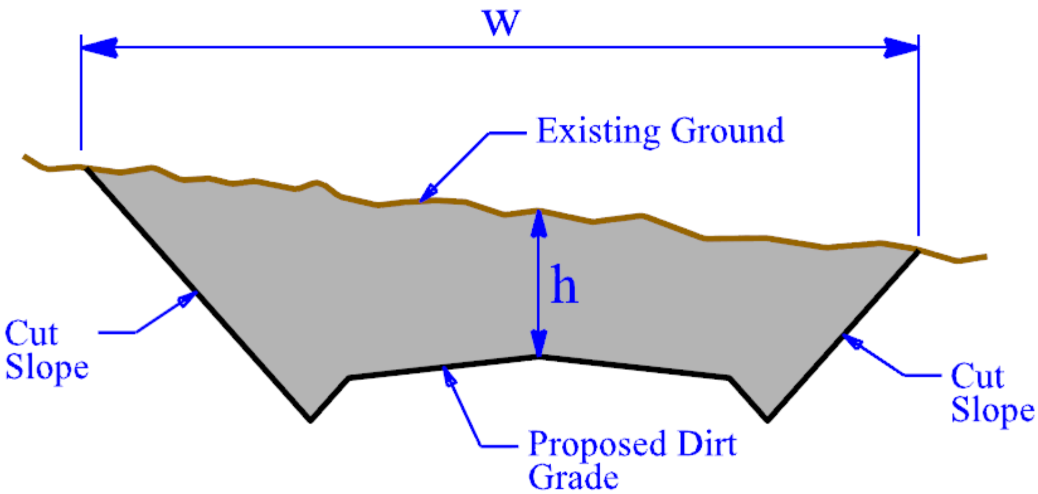

Figure F-13. End-area elements.