Section 3-03
Vertical Alignment

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VERTICAL ALIGNMENT

General: The vertical alignment of a roadway is controlled by design speed, topography, traffic volumes, highway functional classification, sight distance, horizontal alignment, vertical clearances, drainage, economics, and aesthetics. The vertical alignment (or profile) of the roadway is typically established at the point of superelevation rotation. This point is used because it is not affected by the change in elevation when the road is rotated for horizontal curves.

Maximum Grades: Maximum grades to be used for vertical alignment design are dependent on the functional classification, rural or urban designation, design speed, and terrain. Terrain types are based on the local topography and can be either Level, Rolling, or Mountainous. Maximum grade tables for each functional classification can be found in the *AASHTO A Policy on Geometric Design of Highways and Streets 2018*. WYDOT maximum grades to be used in design are defined in the *WYDOT Design Guides.*

Establishing Vertical Alignment: The vertical alignment of a new roadway is established by locating all of the points of intersection (VPIs), at which the grade of the roadway must change. The VPIs are then connected by straight lines to create the tangent portions of the vertical alignment. Vertical curves appropriate for the design speed of the roadway are then added to the tangent alignment to gradually effect the required change in grade at the VPIs. Vertical curves are used to provide a smooth transition between roadway grades. A vertical curve is composed of a parabolic curve that provides a constant rate of change of grade. A parabolic curve "flattens" the vertical alignment at the top or bottom of a hill to increase the driver's sight distance.

There are several factors that will influence the vertical alignment of a roadway:
1) Matching the proposed grade with the existing terrain will reduce the depths of cuts and fills and make it easier to balance the volume of cut material with fill material, which minimizes the need for borrow sources or waste areas.
2) The vertical alignment should not exceed maximum grades, nor should it interfere with existing or proposed drainage structures.
3) The vertical alignment will have to meet fixed elevations to accommodate passing over, passing under, or intersecting with other roadways or railroads.
4) The vertical curve should provide sufficient sight distance.
5) The final vertical alignment should result in an aesthetically pleasing roadway that flows smoothly, fitting the surrounding terrain.

Vertical Curve Geometry: The grade line G is always expressed in terms of a percentage. When the grade rises in the direction of increasing stationing, G is positive. When the grade falls or slopes downward, G is negative. When a grade changes to a grade of less slope, such as over the top of a hill, the vertical curve is called a crest. When a grade increases in slope, such as through a valley, the vertical curve is a sag. Crests and sags are the two types of vertical curves. The grade lines intersect one another at the vertical point of intersection (VPI). The slope of the first grade line is labeled G_1 and the slope of the second grade line is labeled G_2. The algebraic difference of grades is expressed as A. Refer to EXHIBIT 1 to see the different types of vertical curves.
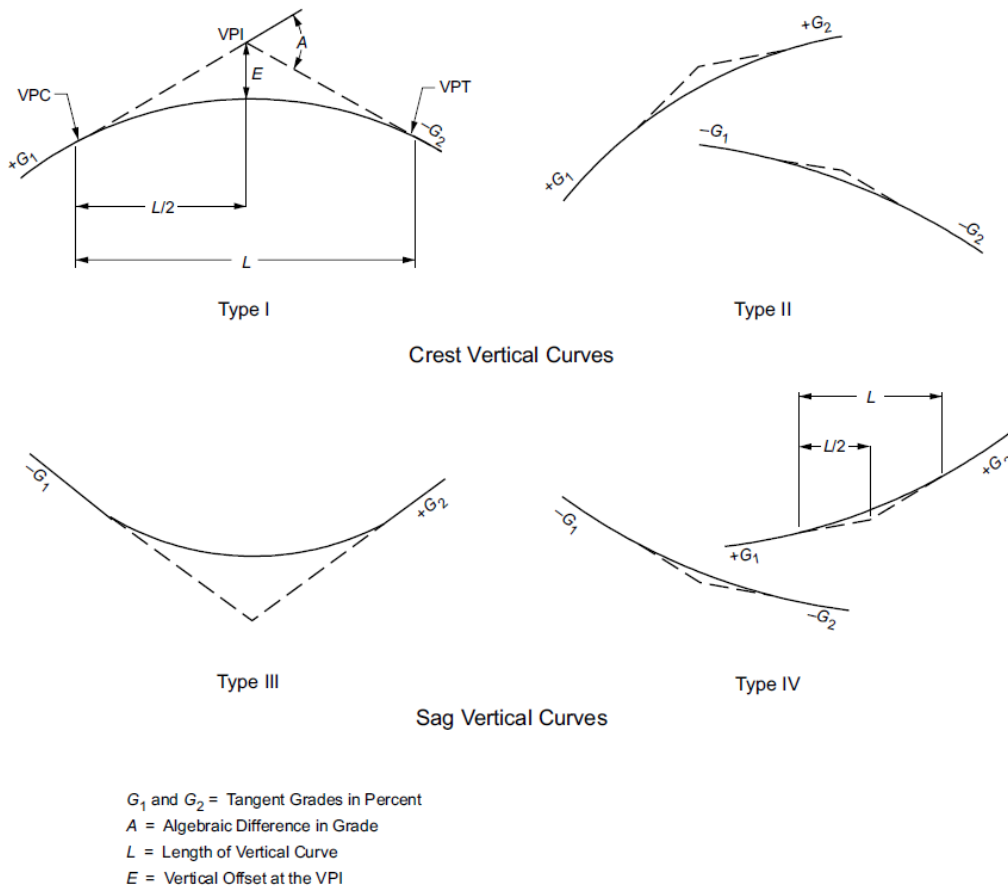

Figure 3-34. Types of Vertical Curves

EXHIBIT 1, TYPES OF VERTICAL CURVES FIGURE

The vertical curve begins at the vertical point of curvature (VPC) point and ends at the vertical point of tangency (VPT) point. The length (L) of the vertical curve is the projection of the curve onto a horizontal surface. All measurements for both symmetrical and unsymmetrical vertical curves are made on the horizontal or vertical planes, **NOT** along the profile grade. For computational purposes the length (L) of a vertical curve is not the distance along the curve. The length of the curve is the difference in stationing between the ends of the curve, regardless of the tangent gradients. In most cases, the vertical curve will have equal-tangent lengths (i.e. the VPI is located midway between the VPC and the VPT), this is called a symmetrical curve.

The high point in a crest curve and the low point in a sag curve are defined by a horizontal distance (x) from the VPC point. The elevation at the high or low point (y) is the vertical distance above a datum. The high and low points are important locations on a vertical curve. For example, the high point on a crest curve must be identified to allow for minimum overhead clearances to an overpass or utility line, and the low point on a sag curve must be identified to allow for minimum amount of cover over drainage structures. Refer to EXHIBIT 2 for an illustration of the components of a typical crest and sag vertical curve.
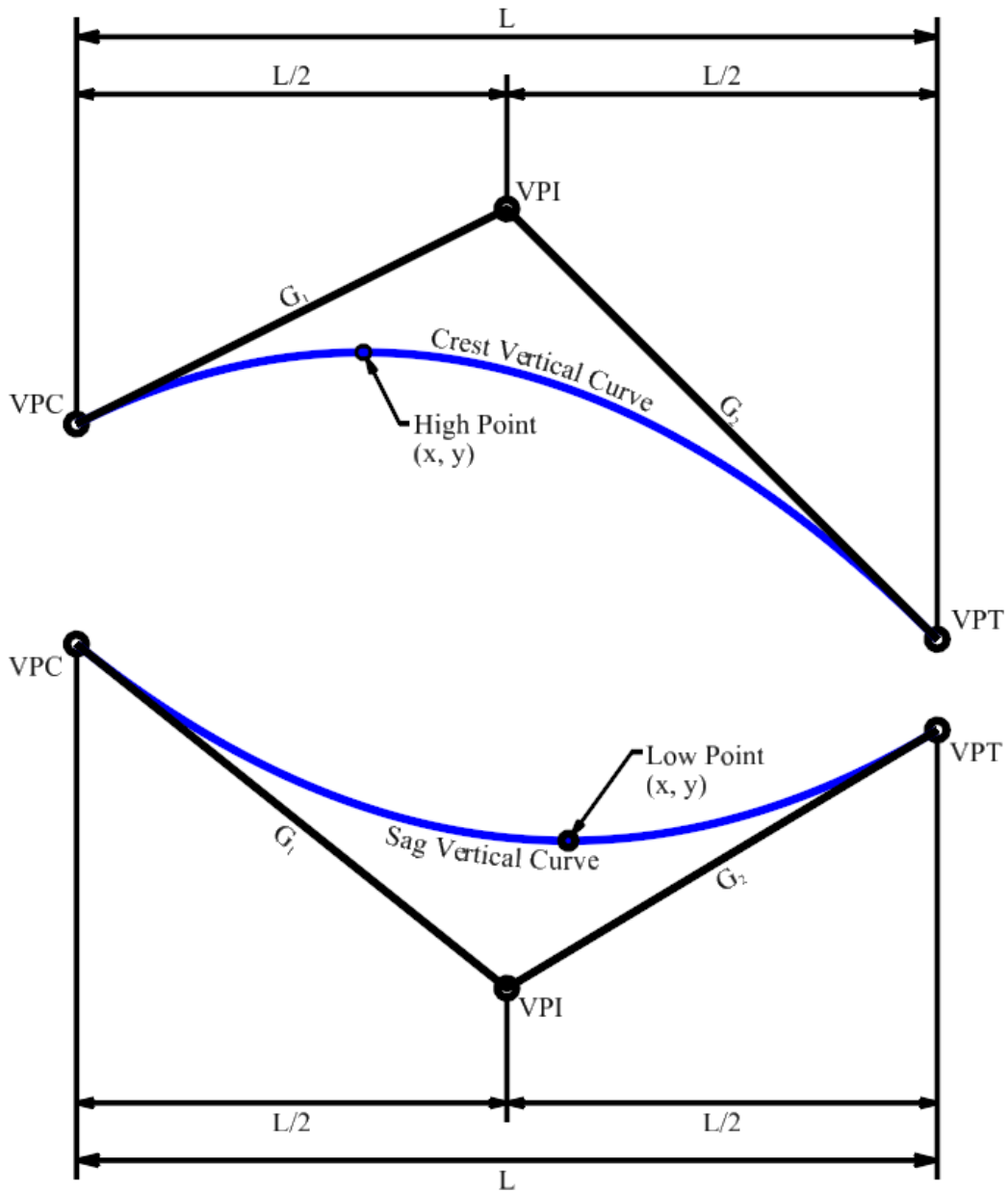
EXHIBIT 2, CREST AND SAG VERTICAL CURVES FIGURE

Vertical curve calculations can be derived from the General Parabolic Equation or by the Tangent Offset Method. The General Parabolic Equation is presented in the procedures used in this document.

General Vertical Curve Equations: The general equation for a parabola is:

$$y = ax^2 + bx + c$$

Where y is the elevation at any point along the parabola located a distance x from the VPC point. The values for a, b, and c are constants that can be calculated from A, G_1, and L as follows:

The slope of a tangent at the beginning of the curve (b) is equal to G_1. The distance of the VPC above or below a vertical datum is equal to c.

To calculate a, first note that the slope of the curve at any point is determined by the first derivative of the parabolic equation:

$$\frac{dy}{dx} = 2ax + b$$

The rate of change of grade is determined by the second derivative of the parabolic equation:

$$\frac{d^2y}{dx^2} = 2a$$

The rate of change of grade (2a) can also be written as A/L.

$$2a = \frac{A}{L}; \quad or \ a = \frac{A}{2L}$$

Since A is the algebraic difference of grade:

$$A = G_2 - G_1$$

The parabolic equation can be rewritten as:

$$y = \frac{A}{2L} * x^2 + G_1 * x + VPC_{ELEV} = \frac{(G_2 - G_1)}{2L} * x^2 + G_1 * x + VPC_{ELEV}$$

At the high or low point of a parabola, the slope is equal to zero. To solve for the x and y value at the high or low point of a vertical curve, use the following set of equations:

$$\frac{dy}{dx} = 2ax + b; \quad or \ Slope = 2ax + G_1$$

Since the slope at the high or low point is equal to zero:

$$0 = 2ax + G_1$$

$$2ax = -G_1$$

$$x = -\frac{G_1}{2a}; \quad since \ a = \frac{A}{2L}; \quad x = -\frac{G_1}{2(A/2L)} = -\frac{G_1 L}{A}$$

Stationing:

$$Sta.VPC = Sta.VPI - L/2$$

$$Sta.VPT = Sta.VPI + L/2$$

Example Problem:

Given: Sta. VPC = 30+30   **Find:** The elevations and
                 G₁ = -3.2%   stations at the low point, VPI,
                 L = 300 ft   and VPT.
                 G₂ = 1.8%
                 VPC Elev. = 4165.92 ft

Solution:

$$A = G_2 - G_1 = 1.8\% - (-3.2\%) = 5.0\% = 0.05$$

$$Sta.VPI = Sta.VPC + L/2 = 3030.00 + 300/2$$
$$= 3180.00 \ or \ \mathbf{31 + 80.00}$$

$$Sta.VPT = Sta.VPI + L/2 = 3180.00 + 300/2$$
$$= 3330.00 \ or \ \mathbf{33 + 30.00}$$

$$VPI \ Elev. = VPC \ Elev. + L/2 * G_1 = 4165.92 + 300/2 * (-0.032)$$
$$= \mathbf{4161.12 \ ft}$$

$$VPT \ Elev. = VPI \ Elev. + L/2 * G_2 = 4161.12 + 300/2 * (0.018)$$
$$= \mathbf{4163.82 \ ft}$$

Low point location:

$$x = -\frac{G_1 L}{A} = -\frac{(-0.032) * 300}{0.05} = \mathbf{192.00\ ft}$$

$$Low\ pt.\ Sta. = Sta.\ VPC + x = 3030.00 + 192.00$$
$$= 3222.00\ or\ \mathbf{32 + 22.00}$$

$$y = \frac{(G_2 - G_1)}{2L} * x^2 + G_1 * x + VPC_{ELEV}$$

$$y = \frac{(0.018 - (-0.032))}{2 * 300} * (192)^2 - 0.032 * 192 + 4165.92$$
$$= \mathbf{4162.85\ ft}$$

Locating Points on Vertical Curves: The elevation of any point located at a distance x from the VPC along the curve can be determined by utilizing the vertical curve formulas.
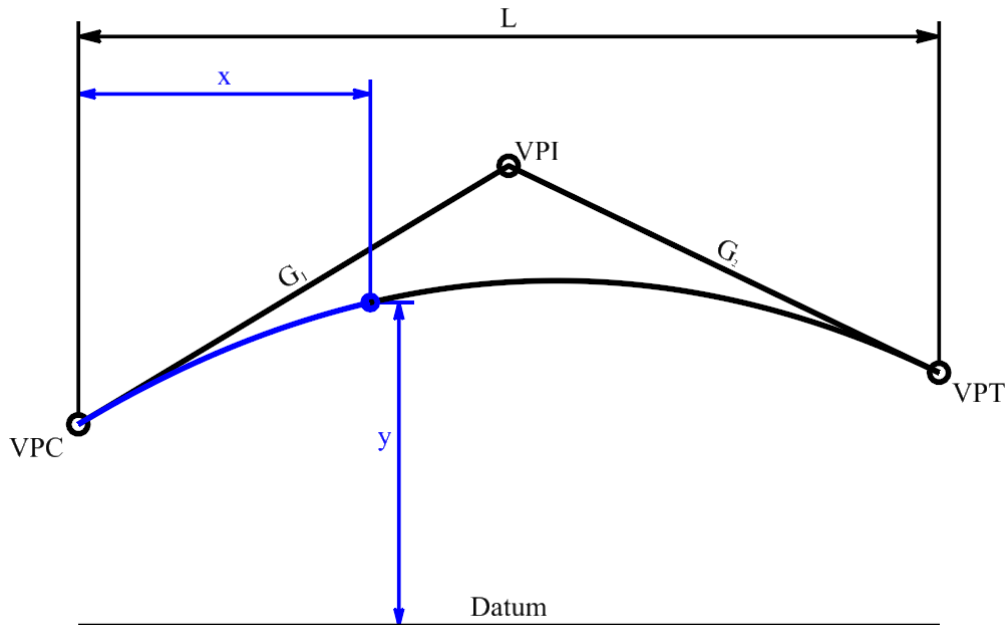
EXHIBIT 3, POINT ON A VERTICAL CURVE FIGURE

The horizontal distance (x) from the VPC to the desired station is the first determination. Next, the Curve Elevation (y) is calculated:

$$y = \frac{A}{2L} * x^2 + G_1 * x + VPC_{ELEV}$$

An elevation along the tangent from the VPC to the VPI can be calculated by the following formula:

$$Tangent\ Elevation = G_1 * x + VPC_{ELEV}$$

Tangent elevations from the VPI to the VPT are determined by the following formula:

$$Tangent\ Elevation = G_2 * (x - L/2) + VPI_{ELEV}$$

Stopping Sight Distance on Vertical Curves: The major control for vertical curve design is providing sight distance greater than or equal to the minimum stopping sight distance for a given design speed. Minimum desirable lengths of crest vertical curves determined by sight distance requirements are generally considered adequate for standards of safety, comfort and appearance. Grade changes should be designed within tolerable limits when considering comfort. In cases of sag vertical curves, the entire curve can be visible during the day, but sight distance is limited by headlights at night.

Table 3-35. Design Controls for Crest Vertical Curves Based on Stopping Sight Distance

U.S. Customary				Metric			
Design Speed (mph)	Stopping Sight Distance (ft)	Rate of Vertical Curvature, Ka		Design Speed (km/h)	Stopping Sight Distance (m)	Rate of Vertical Curvature, Ka	
		Calculated	Design			Calculated	Design
15	80	3.0	3	20	20	0.6	1
20	115	6.1	7	30	35	1.9	2
25	155	11.1	12	40	50	3.8	4
30	200	18.5	19	50	65	6.4	7
35	250	29.0	29	60	85	11.0	11
40	305	43.1	44	70	105	16.8	17
45	360	60.1	61	80	130	25.7	26
50	425	83.7	84	90	160	38.9	39
55	495	113.5	114	100	185	52.0	52
60	570	150.6	151	110	220	73.6	74
65	645	192.8	193	120	250	95.0	95
70	730	246.9	247	130	285	123.4	124
75	820	311.6	312				
80	910	383.7	384				

ᵃ Rate of vertical curvature, K, is the length of curve per percent algebraic difference in intersecting grades (A), $K = L/A$.

EXHIBIT 4, DESIGN CONTROLS FOR CREST VERTICAL CURVES TABLE

Minimum Vertical Curve Length for Stopping Sight Distances (Crest Curves): Crest curve lengths are generally based on stopping sight distances. The required stopping distance (S) is a function of velocity and can be found in EXHIBIT 4. Since it is not known at the time, usually curve length (L) is calculated for both the S < L and S > L. S < L is conservative and used for most design. S > L is used only in situations with severe geometric constraints, such as low speed, urban designs. The constants used are the driver's eye above the roadway surface (h_1=3.5 ft) and specific heights of objects (h_2=2.0 ft) and are found in the *AASHTO A Policy on Geometric Design of Highways and Streets*.

The equations are:

$$L = \frac{AS^2}{100(\sqrt{2h_1} + \sqrt{2h_2})^2} \quad [S < L]$$

$$L = 2S - \frac{200(\sqrt{h_1} + \sqrt{h_2})^2}{A} \quad [S > L]$$

By direct substitution with the constant values for h_1=3.5 ft and h_2=2.0 ft, the simplified formulas become:

$$L = \frac{AS^2}{2158} \quad [S < L]$$

$$L = 2S - \frac{2158}{A} \quad [S > L]$$

Design of Crest Curves Using K-value: A simple and convenient method of choosing a stopping sight distance for a crest vertical curve is expressing the length of vertical curve per percent change in A. The horizontal distance needed to make a one percent change in gradient is a measure of curvature, K, and is the ratio of the curve length, L, to the grade difference, A:

$$K = \frac{L}{A} = \frac{L}{|G_2 - G_1|}$$

$$L_{min} = KA = K * |G_2 - G_1|$$

The *AASHTO A Policy on Geometric Design of Highways and Streets* contains design tables and graphs for determining the stopping sight distance and minimum length of crest vertical curves. The measure of curvature, K, can be found directly in EXHIBIT 4. Alternatively using EXHIBIT 5, the general procedure is to select one of the graph lines based on the speed or the K-value and read the curve length corresponding to the grade difference, A.
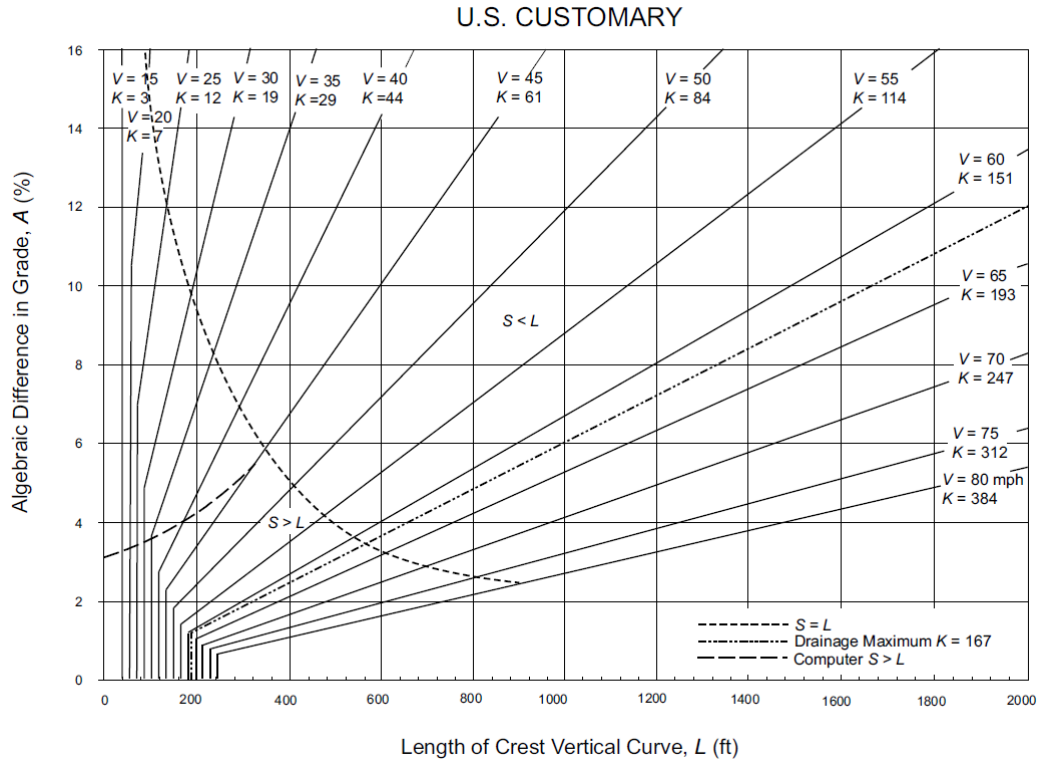
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Figure 3-36. Design Controls for Crest Vertical Curves—Open Road Conditions

EXHIBIT 5, DESIGN CONTROLS FOR CREST VERTICAL CURVES GRAPH

Design of Sag Curves: The design of sag vertical curves must take into consideration many factors including: headlight sight distance, passenger comfort, drainage control, and general appearance. Sag curves are usually controlled by headlight sight distances. Because headlights are pointed downwards, the distance of highway ahead that is lighted on a sag curve is less than on a level road. The equations for curve length based on stopping distance and based on assumed headlight height and angle, are as follows:

$$L = \frac{AS^2}{400 + 3.5S} \quad [S < L]$$

$$L = 2S - \frac{400 + 3.5S}{A} \quad [S > L]$$

K-values can be found in EXHIBIT 6. Alternatively, curve length can be read directly off of EXHIBIT 7.

Table 3-37. Design Controls for Sag Vertical Curves

U.S. Customary				Metric			
Design Speed (mph)	Stopping Sight Distance (ft)	Rate of Vertical Curvature, K[a]		Design Speed (km/h)	Stopping Sight Distance (m)	Rate of Vertical Curvature, K[a]	
		Calculated	Design			Calculated	Design
15	80	9.4	10	20	20	2.1	3
20	115	16.5	17	30	35	5.1	6
25	155	25.5	26	40	50	8.5	9
30	200	36.4	37	50	65	12.2	13
35	250	49.0	49	60	85	17.3	18
40	305	63.4	64	70	105	22.6	23
45	360	78.1	79	80	130	29.4	30
50	425	95.7	96	90	160	37.6	38
55	495	114.9	115	100	185	44.6	45
60	570	135.7	136	110	220	54.4	55
65	645	156.5	157	120	250	62.8	63
70	730	180.3	181	130	285	72.7	73
75	820	205.6	206				
80	910	231.0	231				

[a] Rate of vertical curvature, K, is the length of curve (m) per percent algebraic difference intersecting grades (A), $K = L/A$.

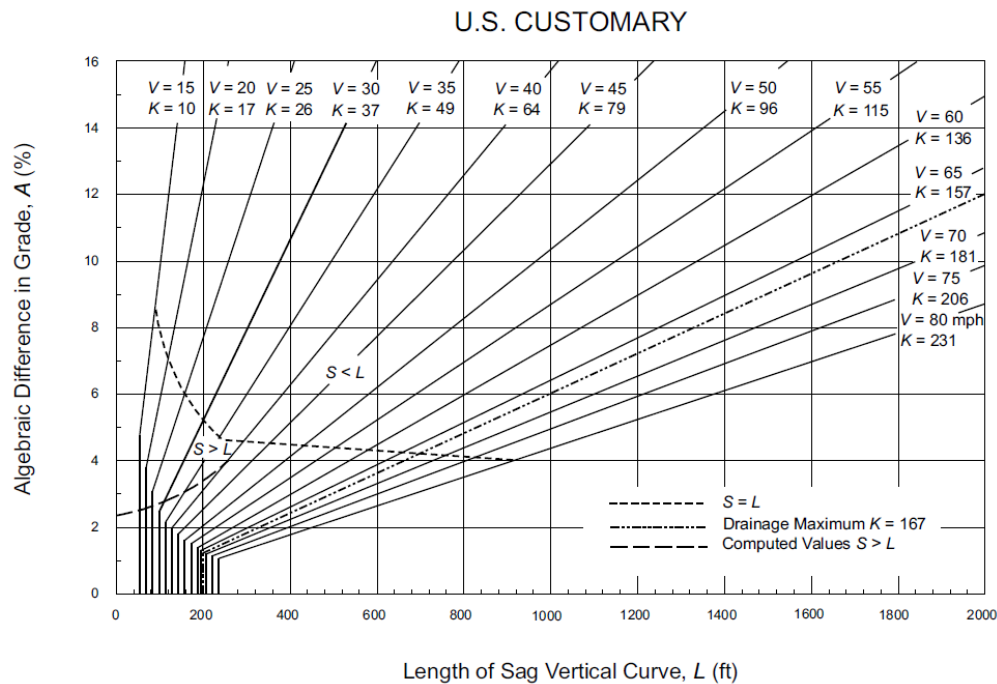**EXHIBIT 6, DESIGN CONTROLS FOR SAG VERTICAL CURVES TABLE**

Figure 3-37. Design Controls for Sag Vertical Curves—Open Road Conditions

EXHIBIT 7, DESIGN CONTROLS FOR SAG VERTICAL CURVES GRAPH

In cases where the roadway is illuminated, comfort usually becomes the limiting factor due to the combination of the gravitational and centrifugal forces acting in the same direction. Because of this, the length of a sag vertical curve should be about half the distance required to furnish stopping sight distance on crest vertical curves.

Be aware that the design controls for sag vertical curves differ from those for crests, and separate design values are needed. The *AASHTO A Policy on Geometric Design of Highways and Streets* contains design tables and graphs for determining the stopping sight distance and minimum length of sag vertical curves, similar to the crest curves. As with the crest curves, the general procedure is to select one of the graph lines based on the speed or the K-value and read the curve length corresponding to the grade difference, A.

As of 2016, the FHWA no longer considers the length of sag vertical curves as a controlling design criterion. Even so, it is good practice to continue calculating minimum curve length, and accept $L < L_{min}$ only under severe geometric constraints.

As shown in EXHIBIT 8, sight distance at under crossings is also an important consideration. Stopping sight distance should always be obtained when crossing under a structure. By using the vertical clearance under the structure for the variable, C, the simplified curve formulas used for sag curve designs are:

$$L = \frac{AS^2}{800(C - 5)} \quad [S < L]$$

$$L = 2S - \frac{800(C - 5)}{A} \quad [S > L]$$
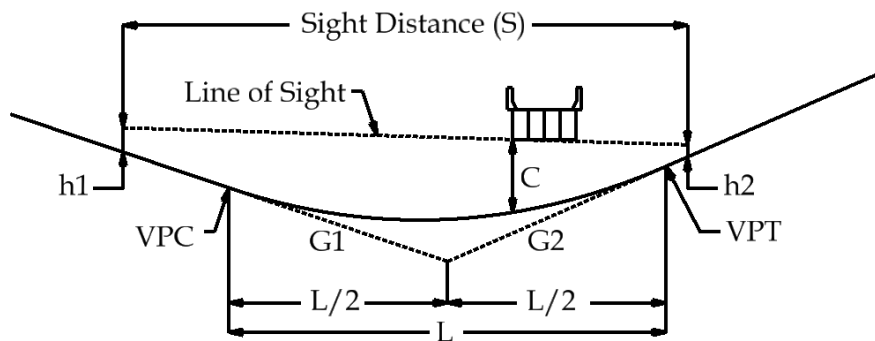
EXHIBIT 8, SIGHT DISTANCE AT UNDERCROSSINGS FIGURE

Unsymmetrical Vertical Curves: This type of vertical curve is utilized only when a standard symmetrical vertical curve cannot be made to fit the terrain. Essentially, the vertical curve consists of two unequal tangent lengths. The horizontal distance from the VPC to the VPI is not equal to the horizontal distance from the VPI to the VPT. The two curves are tangent, so the end of the first curve is also the beginning of the second. This meeting point is referred to as the point of compound vertical curvature (CVC). Refer to EXHIBIT 9 for an illustration of an unsymmetrical vertical curve.

An unsymmetrical vertical curve is comprised of two symmetrical curves adjacent to each other. The calculations required to determine the curve and tangent elevations for each curve are the same as for a single symmetrical curve. However, an additional grade line needs to be determined. A line drawn tangent to the curve at the CVC point is equal to G_2 for the first vertical curve and G_1 for the second. This grade line is parallel to the slope of the grade from the VPC of the first curve and the VPT of the second. The slope of this grade line is determined by the following formula:

$$G = (VPT_{ELEV} - VPC_{ELEV})/L$$
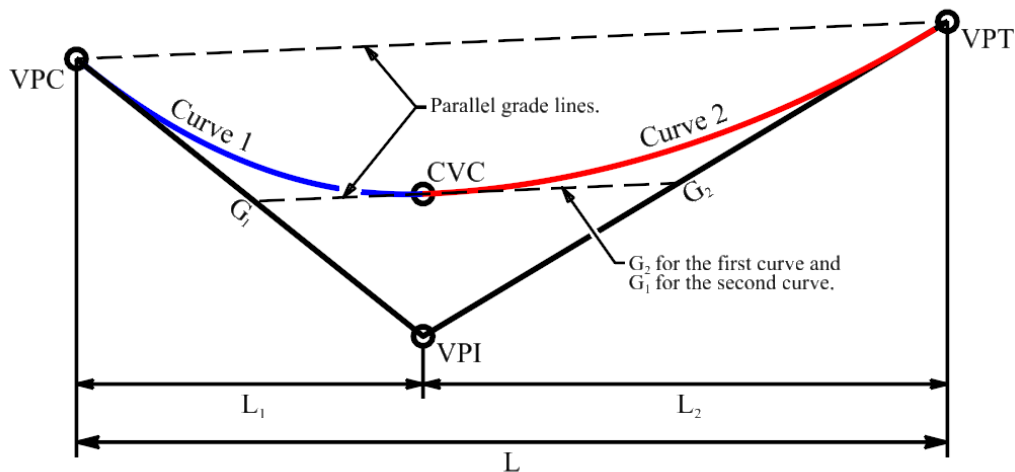
EXHIBIT 9, UNSYMMETRICAL VERTICAL CURVES FIGURE

For Curve 1, the VPI was set halfway between the VPC and the VPI of the unsymmetrical curve. For Curve 2, the VPI was set halfway between the VPI and the VPT. Connecting the VPI of Curve 1 with the VPI of Curve 2 established G_2 for Curve 1 and G_1 for Curve 2.

The K-values of the two curves (K_1 and K_2) are different from the K-value of a single symmetric curve. Both K_1 and K_2 must be adequate for the design speed V. Sight distances near the CVC should be checked graphically, with eye height (h_1=3.5 ft) on Curve 1 and object height (h_2=2.0 ft) on Curve 2.

Passing Sight Distance on Crest Vertical Curves: The eye heights and object heights for passing sight distance differs from those for stopping distances and can be found in the *AASHTO A Policy on Geometric Design of Highways and Streets*. These values can be substituted into the vertical curve formulas to obtain the required vertical curve length for passing. However, extremely long vertical curves are required to furnish passing sight distance, which can result in extensive excavation or embankment cost and difficulty in making the curve fit the terrain. It is not often practical to furnish crest vertical curve lengths which provide adequate passing sight distance. EXHIBIT 10 shows K-values for passing sight distances on vertical crest curves.

Table 3-36. Design Controls for Crest Vertical Curves Based on Passing Sight Distance

U.S. Customary			Metric		
Design Speed (mph)	Passing Sight Distance (ft)	Rate of Vertical Curvature, K^a Design	Design Speed (km/h)	Passing Sight Distance (m)	Rate of Vertical Curvature, K^a Design
20	400	57	30	120	17
25	450	72	40	140	23
30	500	89	50	160	30
35	550	108	60	180	38
40	600	129	70	210	51
45	700	175	80	245	69
50	800	229	90	280	91
55	900	289	100	320	119
60	1000	357	110	355	146
65	1100	432	120	395	181
70	1200	514	130	440	224
75	1300	604			
80	1400	700			

[a] Rate of vertical curvature, K, is the length of curve per percent algebraic difference in intersecting grades (A), $K = L/A$.

EXHIBIT 10, PASSING SIGHT DISTANCE ON CREST VERTICAL CURVES TABLE

Climbing Lanes: If long sections of steep grades exist in a roadway section, the grades should be reviewed to see if climbing lane requirements are met. Free flow operation of two-lane highways can be adversely affected by heavily loaded vehicle traffic operating on grades of sufficient length to result in speeds that could impede following vehicles. The term "critical length of grade" defines the

maximum length of a particular grade that a truck can operate at an appropriate speed. Climbing lanes are an added lane for vehicles moving slowly uphill so that other vehicles using the normal lane to the right of the centerline are not delayed.

The following 3 criteria should be satisfied to justify a climbing lane:
1. Upgrade traffic flow rate in excess of 200 vehicles per hour
2. Upgrade truck flow rate in excess of 20 vehicles per hour
3. One of the following conditions exists:
   - A 10 mph or greater speed reduction is expected for typical heavy truck
   - Level of service E or F exists on the grade
   - A reduction of 2 or more levels of service is experienced when moving from the approach segment to the grade

The *AASHTO A Policy on Geometric Design of Highways and Streets* should be referenced for detailed explanation and procedure examples for climbing lane analysis. Recommendations will usually be provided by the Traffic Program for locations of proposed climbing lanes.

Vertical Alignment Design Considerations:

1. Minimum vertical curve lengths should not be used routinely in design. The roadway should be designed using above minimum values when feasible, taking into account safety, function, aesthetics, availability of right of way, economics, etc.

2. Roadways should be designed in order to fit the contours of the natural surroundings. The roadway should not appear abnormal or unnatural by having a vertical alignment that is inconsistent with the existing terrain.

3. One of the major considerations in setting the vertical alignment on many projects is balancing excavation and embankment. The vertical alignment can also be adjusted to minimize the required amount of haul.

4. The same design speed should be used throughout a particular section of roadway. **Isolated vertical curves cannot be posted for reduced speeds, even though horizontal curves are occasionally signed for lower speeds.** Transitions should be provided in areas where the design speed changes to allow the driver to gradually become accustomed to the changed condition.

5. Changes in grade are best accomplished by using symmetrical vertical curves. Unsymmetrical vertical curves should only be used in special situations.

6. Vertical alignment is often controlled by factors such as meeting existing intersections or access points, obtaining minimum vertical clearance over an intersecting roadway or providing minimum cover over pipe culverts or utilities, etc.

7. Drainage considerations on many projects frequently control the vertical alignment. For example, storm drain design and vertical alignment design are closely interrelated. The designer should take care to provide vertical curve lengths and locations so as to avoid "flat spots" and areas of poor drainage along the roadway. To assure proper drainage in curb and gutter sections, the roadway grade preferably should not be less than 0.5% and never flatter than 0.3%.